Kuwait University — Math 101 — Date: December 21, 2008
Dept. of Math. & Comp. Sci. — Second Exam — Duration: 90 minutes

Calculators, mobile phones, pagers and all other mobile communication equipments are not allowed

Answer the following questions:

1. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$, at $x=0$, where $2y+\sin(xy)=1$. (4 pts.)

2. The volume of a cube is increasing at a rate of $30\ \text{cm}^3/\sec$. How fast is the total surface area of the cube increasing when the length of each edge of the cube is 10 cm long? (4 pts.)

3. Use differentials to approximate $\sqrt[3]{1.03}$. (3 pts.)

4. Find the absolute extrema, if any, of $f(x)=\sin x+\cos x$ on $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$. (3 pts.)

5. Use Rolle's Theorem to show that the equation $x^7+4x^3+x-2=0$ cannot have two different roots. (3 pts.)

6. Let $f(x)=\dfrac{(x+3)^2}{x}$.

   (a) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find the local extrema of f, if any.

   (b) Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the points of inflection, if any.

   (c) Sketch the graph of f. (8 pts.)

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1. At $x=0$, $y=\frac{1}{2}$. Differentiate twice w.r.t. x. $2y'+(y+xy')\cos(xy)=0$ & $2y''-(y+xy')^2\sin(xy)+(2y'+xy'')\cos(xy)=0 \implies \boxed{\left.\frac{dy}{dx}\right|_{(0,\frac{1}{2})}=-\frac{1}{4}}$ & $\boxed{\left.\frac{d^2y}{dx^2}\right|_{(0,\frac{1}{2})}=\frac{1}{4}}$.

2. $V=l^3 \implies \frac{dV}{dt}=3l^2\frac{dl}{dt} \implies \left.\frac{dl}{dt}\right|_{l=10}=\frac{1}{10}$ cm/sec. $S=6l^2 \implies \frac{dS}{dt}=12l\frac{dl}{dt} \implies \boxed{\left.\frac{dS}{dt}\right|_{l=10}=12\ \text{cm}^2/\sec}$

3. Let $f(x)=\sqrt[3]{x}$ and $x_0=1$, then $f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$ and , $\Delta x=0.03$. So, $\sqrt[3]{1.03}=f(1.03)\simeq f(1)+f'(1)\Delta x=1+\frac{1}{3}(0.03)=\boxed{1.01}$.

4. $f'(x)=\cos x-\sin x$. The critical number of f in $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ is $x=\frac{5\pi}{4}$, when $f'(x)=0$.

x	$f(x)$	Classification of x
$\frac{\pi}{2}$	1	end point
$\frac{5\pi}{4}$	$-\sqrt{2}$	critical number
$\frac{3\pi}{2}$	-1	end point

$f\left(\frac{\pi}{2}\right)=1$ is the absolute maximum of f in $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$,
$f\left(\frac{5\pi}{4}\right)=-\sqrt{2}$ is the absolute minimum of f in $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$.

5. Suppose that a and b are two different real roots of the equation ($a<b$, say). Let $f(x)=x^7+4x^3+x-2$ and consider the interval $[a,b]$. Since f is a polynomial, then f is continuous on $[a,b]$ and differentiable on (a,b). $f'(x)=7x^6+12x^2+1$. Since a,b are roots of f, then $f(a)=0=f(b)$. From Rolle's Theorem, $\exists\ c\in(a,b)$ such that $f'(c)=0$, but $f'(c)=7c^6+12c^2+1>0$. Thus, f cannot have two different real roots.

6. Domain $f=\mathbb{R}-\{0\}$. The point $(-3,0)$ lies on the curve. $\boxed{f'(x)=\frac{x^2-9}{x^2}}$ & $\boxed{f''(x)=\frac{18}{x^3}}$.

   (a) The critical numbers of f are $x=-3$ and $x=3$, when $f'(x)=0$. At $x=0$, f'does not exist.

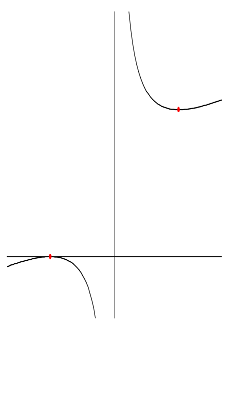
I	$(-\infty,-3)$	$(-3,0)$	$(0,3)$	$(3,\infty)$
sign of $f'(x)$	$+$	$-$	$-$	$+$
Conclusion	↗	↘	↘	↗

   f is increasing on $(-\infty,-3)$ and $(3,\infty)$. f is decreasing on $(-3,0)$ and $(0,3)$. $f(-3)=0$ is a local maximum of f. $f(3)=12$ is a local minimum of f.

   (b) $f''(0)$ does not exist.

I	$(-\infty,0)$	$(0,\infty)$
sign of $f''(x)$	$-$	$+$
Concavity	CD	CU

   (c)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